Mathematics Unit Planning Guide and Resources

PAJARO VALLEY UNIFIED SCHOOL DISTRICT

This guide explains components of the mathematics unit planning template.

PVUSD Mathematics: Unit of Study Planner Guide

Suggested Pacing of the Unit

Unit _____of _____

_____ Days

Identifying Information for the Unit and list of instructional resources

GRADE OR COURSE	STANDARDS
	FLUENCY TARGET FOR GRADE LEVEL
UNIT TITLE	
INSTRUCTIONAL RESOURCES: MATERIALS, TOOLS, TECHNOLOGY	
FULL TEXT - MATHEMATICS CONTENT STANDARDS	

STANDARDS

Include the grade, domain, and standard. For example, K.C.C.1 indicates Kindergarten, Counting and Cardinality, Standard 1.

GRADE LEVEL FLUENCY TARGET

Wherever the word *fluently* appears in a content standard, the word means quickly and accurately. It means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself. A key aspect of fluency in this sense is that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

Grade	Standard	Expected Fluency
K	K.OA.A.5	Add/Subtract within 5
1	1.OA.C.6	Add/Subtract within 10
2	2.OA.B.2 2.NBT.B.5	Add/Subtract within 20 (Know single digit sums from memory) Add/Subtract within 100
3	3.OA.C.7 3.NBT.A.2	Multiply/Divide within 100 (Know single digit products from memory) Add/Subtract within 1000
4	4.NBT.B.4	Add/Subtract within 1,000,000
5	5.NBT.B.5	Multi-digit multiplication
6	6.NS.B.2 6.NS.B.3	Multi-digit division Multi-digit decimal operations
7	7.NS.A.1,2 7.EE.B.3 7.EE.B.4	Fluency with rational number arithmetic Solve multistep problems with positive and negative rational numbers in any form Solve one-variable equations of the form $px + q = r$ and $p(x + q) = r$ fluently
8	8.EE.C.7 8.G.C.9	Solve one-variable linear equations, including cases with infinitely many solutions or no solutions Solve problems involving volumes of cones, cylinders, and spheres together with previous geometry work, proportional reasoning and multi-step problem solving in grade 7

FULL TEXT - MATHEMATICS CONTENT STANDARDS

List each standard's grade, domain, and standard and include the full text from the standard. For example:

K.CC.1
Count to 100 by ones and by tens.

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OVERVIEW: 21st CENTURY THEMES AND SKILLS

Units of instruction are developed under the umbrella of 21st Century Themes and Skills. On March 11, 2013, California joined the Partnership for 21st Century Learning (P21). P21 is a national organization based in Washington, D.C. and catalyst in the 21st Century Skills Movement. P21 works with education leaders, the business community, and state and federal policymakers to ensure that the U.S. education system equips students with rigorous content knowledge and the skills they need for college, career, and citizenship.

21st Century Curriculum and Instruction:

- Teaches 21st Century Skills discretely in the context of core subjects and 21st century interdisciplinary themes.
- Focuses on providing opportunities for applying 21st century skills across content areas and for a competency-based approach to learning
- Enables innovative learning methods that integrate the use of supportive technologies, inquiry- and problem-based approaches and higher order thinking skills
- Encourages the integration of community resources beyond school walls

21st Century Themes

Interdisciplinary

- ☐ Global Awareness
- ☐ Financial, Economic, Business, and Entrepreneurial Literacy
- ☐ Civic Literacy
- ☐ Health Literacy
- ☐ Environmental Literacy

21st Century Skills

Learning and Innovation Skills

- ☐ Creativity and Innovation
- ☐ Critical Thinking and Problem Solving
- ☐ Communication and Collaboration

Life and Career Skills

- ☐ Flexibility and Adaptability
- ☐ Initiative and Self-Direction
- ☐ Productivity and Accountability
- ☐ Leadership and Responsibility
- ☐ Cross-Cultural and Social Skills

The following pages explain 21st Century skills and themes in terms of what students should know and be able to do.

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21st Century Themes
Interdisciplinary
- ☐ Global Awareness
- ☐ Financial, Economic, Business, and Entrepreneurial Literacy
- ☐ Civic Literacy
- ☐ Health Literacy
- ☐ Environmental Literacy

21st CENTURY THEMES

Global Awareness
- Using 21st century skills to understand and address global issues
- Learning from and working collaboratively with individuals representing diverse cultures, religions and lifestyles in a spirit of mutual respect and open dialogue in personal, work and community contexts
- Understanding other nations and cultures, including the use of non-English languages

Financial, Economic, Business, and Entrepreneurial Literacy
- Knowing how to make appropriate personal economic choices
- Understanding the role of the economy in society
- Using entrepreneurial skills to enhance workplace productivity and career options

Civic Literacy
- Participating effectively in civic life through knowing how to stay informed and understanding governmental processes
- Exercising the rights and obligations of citizenship at local, state, national and global levels
- Understanding the local and global implications of civic decisions

Health Literacy
- Obtaining, interpreting and understanding basic health information and services and using such information and services in ways that are health enhancing
- Understanding preventive physical and mental health measures, including proper diet, nutrition, exercise, risk avoidance and stress reduction
- Using available information to make appropriate health-related decisions
- Establishing and monitoring personal and family health goals
- Understanding national and international public health and safety issues

Environmental literacy
- Demonstrate knowledge and understanding of the environment and the circumstances and conditions affecting it, particularly as relates to air, climate, land, food, energy, water and ecosystems
- Demonstrate knowledge and understanding of society's impact on the natural world (e.g., population growth, population development, resource consumption rate, etc.)
- Investigate and analyze environmental issues, and make accurate conclusions about effective solutions
- Take individual and collective action towards addressing environmental challenges (e.g., participating in global actions, designing solutions that inspire action on environmental issues)

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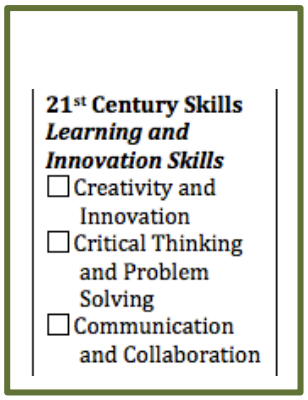
21ST CENTURY LEARNING AND INNOVATION SKILLS



CREATVITY AND INNOVATION

Think Creatively

- Use a wide range of idea creation techniques (such as brainstorming)
- Create new and worthwhile ideas (both incremental and radical concepts)
- Elaborate, refine, analyze and evaluate their own ideas in order to improve and maximize creative efforts

Work Creatively with Others

- Develop, implement and communicate new ideas to others effectively
- Be open and responsive to new and diverse perspectives; incorporate group input and feedback into the work
- Demonstrate originality and inventiveness in work and understand the real world limits to adopting new ideas
- View failure as an opportunity to learn; understand that creativity and innovation is a long-term, cyclical process of small successes and frequent mistakes

Implement Innovations

- Act on creative ideas to make a tangible and useful contribution to the field in which the innovation will occur

CRITICAL THINKING AND PROBLEM SOLVING

Reason Effectively

- Use various types of reasoning (inductive, deductive, etc.) as appropriate to the situation

Use Systems Thinking

- Analyze how parts of a whole interact with each other to produce overall outcomes in complex systems

Make Judgments and Decisions

- Effectively analyze and evaluate evidence, arguments, claims and beliefs
- Analyze and evaluate major alternative points of view
- Synthesize and make connections between information and arguments
- Interpret information and draw conclusions based on the best analysis
- Reflect critically on learning experiences and processes

Solve Problems

- Solve different kinds of non-familiar problems in both conventional and innovative ways
- Identify and ask significant questions that clarify various points of view and lead to better solutions

COMMUNICATION AND COLLABORATION

Communicate Clearly

- Articulate thoughts and ideas effectively using oral, written and nonverbal communication skills in a variety of forms and contexts
- Listen effectively to decipher meaning, including knowledge, values, attitudes and intentions
- Use communication for a range of purposes (e.g. to inform, instruct, motivate and persuade)
- Utilize multiple media and technologies, and know how to judge their effectiveness a priori as well as assess their impact
- Communicate effectively in diverse environments (including multi-lingual)

Collaborate with Others

- Demonstrate ability to work effectively and respectfully with diverse teams
- Exercise flexibility and willingness to be helpful in making necessary compromises to accomplish a common goal
- Assume shared responsibility for collaborative work, and value the individual contributions made by each team member

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21ST CENTURY LIFE AND CAREER SKILLS

Life and Career Skills
- ☐ Flexibility and Adaptability
- ☐ Initiative and Self-Direction
- ☐ Productivity and Accountability
- ☐ Leadership and Responsibility
- ☐ Cross-Cultural and Social Skills

FLEXIBILITY AND ADAPTABABILITY

Adapt to Change
- Adapt to varied roles, jobs responsibilities, schedules and context
- Work effectively in a climate of ambiguity and changing priorities

Be Flexible
- Incorporate feedback effectively
- Deal positively with praise, setbacks and criticism
- Understand, negotiate and balance diverse views and beliefs to reach workable solutions, particularly in multi-cultural environments

INITIATIVE AND SELF-DIRECTION

Manage Goals and Time
- Set goals with tangible and intangible success criteria
- Balance tactical (short-term) and strategic (long-term) goals
- Utilize time and manage workload efficiently

Work Independently
- Monitor, define, prioritize and complete tasks without direct oversight

Be Self-directed Learners
- Go beyond basic mastery of skills and/or curriculum to explore and expand one's own learning and opportunities to gain expertise
- Demonstrate initiative to advance skill levels towards a professional level
- Demonstrate commitment to learning as a lifelong process
- Reflect critically on past experiences in order to inform future progress

SOCIAL AND CROSS-CULTURAL SKILLS

Interact Effectively with Others
- Know when it is appropriate to listen and when to speak
- Conduct themselves in a respectable, professional manner

Work Effectively in Diverse Teams
- Respect cultural differences and work effectively with people from a range of social and cultural backgrounds
- Respond open-mindedly to different ideas and values
- Leverage social and cultural differences to create new ideas and increase both innovation and quality of work

Manage Projects
- Set and meet goals, even in the face of obstacles and competing pressure
- Prioritize, plan and manage work to achieve the intended result

Produce Results
- Demonstrate additional attributes associated with producing high quality products including the abilities to: Work positively and ethically; Manage time and projects effectively; Multi-task; Participate actively, as well as be reliable and punctual; Present oneself professionally and with proper etiquette; Collaborate and cooperate effectively with teams; Respect and appreciate team diversity; Be accountable for results.

Guide and Lead Others

- Use interpersonal and problem-solving skills to influence and guide others toward a goal
- Leverage strengths of others to accomplish a common goal
- Inspire others to reach their very best via example and selflessness
- Demonstrate integrity and ethical behavior in using influence and power

Be Responsible to Others

- Act responsibly with the interests of the larger community in mind

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ENDURING UNDERSTANDINGS

Enduring understandings are a generalization specifying what we want students to come to understand about the big idea(s) of the unit.

Enduring Understandings:

- Synthesize what students should understand – not just know and do – as a result of mastering the standards
- Articulate what students should recall/revisit over the course of their lifetimes
- Assessment of enduring understanding is open ended, complex, and authentic (performance tasks and projects)

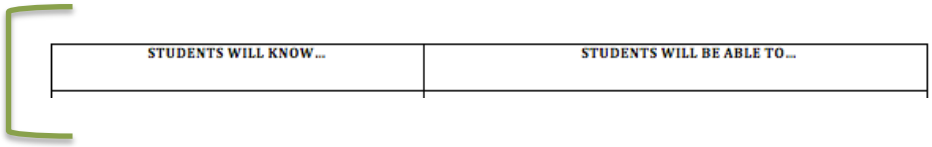
ESSENTIAL QUESTIONS

Essential Questions a provocative and arguable question designed to guide inquiry into the big ideas

Essential Questions:

- Helps students develop and deepen their understandings
- Have no one obvious right answer
- Raise other important questions, often across subject-area boundaries
- Address the philosophical or conceptual foundations of a discipline
- Recur naturally
- Are framed to provoke and sustain student interest

Write learning objectives in terms of what you can and cannot see/hear



STUDENTS WILL KNOW vs. STUDENTS WILL BE ABLE TO

The verbs used to describe student learning objectives fall into two categories.

WHAT STUDENTS WILL KNOW

- Internal cognitive processes we cannot see or hear
- Objectives with verbs such as know, understand, and learn
- *Example:* ***Understand*** *the relationship between numbers and quantities*

WHAT STUDENTS WILL BE ABLE TO DO

- Behaviors we can see and/or hear
- Objectives with verbs such as write, solve, count, explain, point, and say
- *Example: When counting objects,* ***say*** *the number names in the standard order*

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Plan support for both native English speakers and English Learners

ESSENTIAL ACADEMIC LANGUAGE	MODES OF COMMUNICATION	SPEAKING AND LISTENING CONTEXTS
	☐ Collaborative ☐ Interpretative ☐ Presentation	☐ Peer (1:1) ☐ Small Group ☐ Whole Group
	LANGUAGE PURPOSES	TEXT TYPES

ESSENTIAL ACADEMIC LANGUAGE

- Mathematical and contextual vocabulary
- Identify the language students need to know in order to learn and express their understanding of the content
- Consider what students need to learn about how English works to understand the academic and real-world context of vocabulary in the lesson

MODES OF COMMUNICATION

- Collaborative: Engagement in dialogue with others (include throughout the unit)
- Interpretive: Comprehension and analysis of written and spoken text (embedded in performance tasks)
- Productive: Creation of oral presentations and written texts (may not be present in every unit)

SPEAKING AND LISTENING CONTEXTS

- The Common Core Anchor Standards for College and Career Readiness include Speaking and Listening
- Each using should include opportunities for students to speak an listen in more than one context

LANGUAGE PURPOSES

- This section addresses *how* language is used in the unit
- Examples: Counting, describing, comparing/contrasting, classifying, justifying, and critiquing

TEXT TYPES

- This section address *what* type of text the language is embedded within
- Examples: Number Talks, counting books, word problems, theorems, textbooks, and online resources

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Identify end-of-unit assessment and note refinements

END OF UNIT SUMMATIVE ASSESSMENT	
END OF UNIT REFLECTION	QUESTIONS FOR REFLECTION

END OF UNIT SUMMATIVE ASSESSMENT

This assessment measures student learning after the unit is delivered. It provides data that describes students' achievement in relation to a clearly defined set of standards. These assessments are given after students have time to practice and learn from their mistakes through formative assessment.

Summative assessments:

- Inform us about the effectiveness of instruction – Results are used to revise unit and lesson plans
- Align instruction, student practice, and formative assessment to the summative instrument
- Balance authentic use of concepts and skills with procedural skills and fluencies
- Are used to report academic proficiency of students, parents, support teams and in official reporting
- Examples: Quizzes, performance tasks, tests, projects, and portfolios

END OF UNIT REFLECTION

Instructional plans are never "finished" – there is always room for refinement based on observations and student data.

- After teaching the unit, reflect on the strengths and weaknesses of the lessons, activities, and assessments.
- Make notes to inform refinement for the next delivery of this unit using the provided and self-identified reflection points

Sample Questions for Reflection:

- How does this lesson reflect the "shifts" of focus, coherence, and rigor?
- How did this lesson support 21st Century Skills?
- How did this lesson reflect academic rigor?
- How did this lesson cognitively engage students?
- How did this lesson engage students in collaborative learning and enhance their collaborative learning skills?

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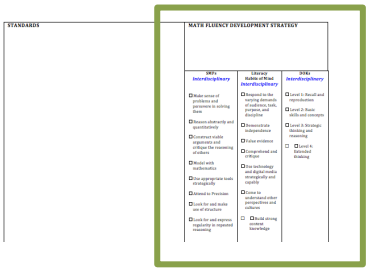
LESSONS WITHIN THE UNIT PLAN

The lessons within the unit are designed to allow students to develop skills and deep, conceptual knowledge to meet the rigor of the common core standards. Several types of lessons are identified in the unit planner.

- Scaffolding Access
- Knowledge-Building
- Practice
- Performance Assessment (aka Authentic Assessment, Performance Task)
- Re-Engagement
- Summative Assessment

Keep in mind:

- Each lesson type could take more than one day.
- Several lesson types may be repeated in the unit. For example, you may have three knowledge-building and five practice lessons within a unit.
- The Performance Assessment lesson could be used as the Summative Assessment for the unit if students have engaged with a sufficient number of formative assessments.



COMPONENTS OF EACH LESSON

STANDARDS

Written as grade, domain, and standard. *For example, K.C.C.1 indicates Kindergarten, Counting and Cardinality, Standard 1.*

MATH FLUENCY DEVELOPMENT STRATEGY

Note the strategies such as games, drills, and/or homework practice in the lesson that will develop the skill(s) within the fluency standard for the grade level.

STANDARDS FOR MATHEMATICAL PRACTICE (SMPs)

The Standards for Mathematical Practice describe the attributes of mathematically proficient students. These standards don't just describe how students should use mathematics: they also provide a vehicle through which students engage with and learn mathematics.

As students move from elementary school through high school, the Standards for Mathematical Practice remain the same. *What changes is the way these standards look as students engage with and master new and more advanced mathematical ideas.* The Standards for Mathematical Practice must be taught as carefully and practiced as intentionally as the Standards for Mathematical Content. Neither should be isolated from the other; impactful mathematics instruction occurs when these two halves of the common core math standards come together in a powerful whole.

The eight SMPs are:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

1. Make sense of problems and persevere in solving them:

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Teachers who are developing students' capacity to "make sense of problems and persevere in solving them" develop ways of framing mathematical challenges that are clear and explicit, and then check in repeatedly with students to help them clarify their thinking and their process.

2. Reason abstractly and quantitatively:

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Teachers who are developing students' capacity to "reason abstractly and quantitatively" help their learners understand the relationships between problem scenarios and mathematical representation, as well as how the symbols represent strategies for solution.

3. Construct viable arguments and critique the reasoning of others

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Teachers who are developing students' capacity to "construct viable arguments and critique the reasoning of others" require their students to engage in active mathematical discourse. This might involve having students explain and discuss their thinking processes aloud, or signaling agreement/disagreement with a hand signal.

4. Model with mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Teachers who are developing students' capacity to "model with mathematics" move explicitly between real-world scenarios and mathematical representations of those scenarios.

5. Use appropriate tools strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Teachers who are developing students' capacity to "use appropriate tools strategically" make clear to students why the use of manipulatives, rulers, compasses, protractors, and other tools will aid their problem solving processes.

6. Attend to Precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Teachers who are developing students' capacity to "attend to precision" focus on clarity and accuracy of process and outcome in problem solving. A teacher might engage his students in a "number talk" in which students use an in/out table and a plotted graph to "guess [the teacher's] number."

7. Look for and make use of structure

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later,

students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Teachers who are developing students' capacity to "look for and make use of structure" help learners identify and evaluate efficient strategies for solution.

8. Look for and express regularity in repeated reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x2 + x + 1), and (x – 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Integrating Standard Eight into classroom practice is not only a matter of planning for lessons that occasion students to look for general methods and shortcuts. It also requires teachers to attend to and listen closely to their students' noticings and "a-ha moments," and to follow those a-ha moments so that they generalize to the classroom as a whole.

LITERACY HABITS OF MIND

Literacy in the common core standards is defined as proficiency in reading, writing, speaking, listening, and language. To support of college and career readiness for all students, literacy is built into the common core standards for all subject areas at every grade level. Including the literacy habits of mind in the unit plan ensures that we attend to developing behaviors in students that support success beyond the classroom walls.

Students who have developed literacy habits of mind:

- **Respond to the varying demands of audience, task, purpose, and discipline**
  - Students adapt their communication in relation to audience, task, purpose, and discipline. They set and adjust purpose for reading, writing, speaking, listening, and language use as warranted by the task. They appreciate nuances, such as how the composition of an audience should affect tone when speaking and how the connotations of words affect meaning. They also know that different disciplines call for different types of evidence (e.g., documentary evidence in history, experimental evidence in science).

- **Demonstrate independence**
  - Students can, without significant scaffolding, comprehend and evaluate complex texts across a range of types and disciplines, and they can construct effective arguments and convey intricate or multifaceted information. Likewise, students are able independently to discern a speaker's key points, request clarification, and ask relevant questions. They build on others' ideas, articulate their own ideas, and confirm they have been understood. Without prompting, they demonstrate command of standard English and acquire and use a wide-ranging vocabulary. More broadly, they become self-directed learners, effectively seeking out and using resources to assist them, including teachers, peers, and print and digital reference materials.

- **Value evidence**
  - Students cite specific evidence when offering an oral or written

interpretation of a text. They use relevant evidence when supporting their own points in writing and speaking, making their reasoning clear to the reader or listener, and they constructively evaluate others' use of evidence.

- **Comprehend as well as critique**
  - Students are engaged and open-minded—but discerning—readers and listeners. They work diligently to understand precisely what an author or speaker is saying, but they also question an author's or speaker's assumptions and premises and assess the veracity of claims and the soundness of reasoning.

- **Use technology and digital media strategically and capably**
  - Students employ technology thoughtfully to enhance their reading, writing, speaking, listening, and language use. They tailor their searches online to acquire useful information efficiently, and they integrate what they learn using technology with what they learn offline. They are familiar with the strengths and limitations of various technological tools and mediums and can select and use those best suited to their communication goals.

- **Come to understand other perspectives and cultures**
  - Students appreciate that the twenty-first-century classroom and workplace are settings in which people from often widely divergent cultures and who represent diverse experiences and perspectives must learn and work together. Students actively seek to understand other perspectives and cultures through reading and listening, and they are able to communicate effectively with people of varied backgrounds. They evaluate other points of view critically and constructively. Through reading great classic and contemporary works of literature representative of a variety of periods, cultures, and worldviews, students can vicariously inhabit worlds and have experiences much different from their own.

- **Build strong content knowledge**
  - Students establish a base of knowledge across a wide range of subject matter by engaging with works of quality and substance. They become proficient in new areas through research and study. They read purposefully and listen attentively to gain both general knowledge and discipline-specific expertise. They refine and share their knowledge through writing and speaking.

DEPTH OF KNOWLEDGE (DOK)

Depth of knowledge refers to the complexity of mental processing required to answer a question, perform a task, or generate a product. It is not:

- A taxonomy (Bloom's)
- The same thing as "difficulty"

Level 1 – Recall and Reproduction

Level 1 includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 – Basic Skills and Concepts

Level 2 includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first

identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 – Strategic Thinking and Reasoning

Level 3 requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 – Extended Thinking

Level 4 requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

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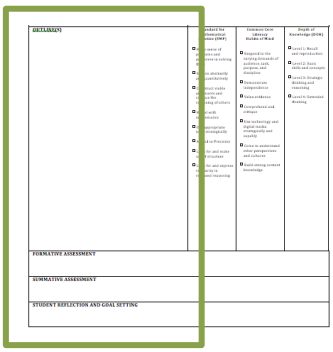

LESSON OUTLINES

Each type of lesson has a different purpose and outcome; the purpose and outcome of each lesson type follows the explanation of Formative Assessment and Student Reflection and Goal Setting.

In this section, you will provide a "sketchy" outline of the lessons – a quick overview of the activities, tasks, etc. Full lesson plans are fleshed out in the actual lesson plan.

FORMATIVE AND SUMMATIVE ASSESSMENT

Formative Assessment

This type of assessment differs from summative assessment in that the purpose is to inform instruction and build metacognition in students. It is considered assessment FOR learning and is not graded. Feedback is given by the instructor in the form of comments and/or questions that guide learning rather than a percentage or letter grade.

Summative Assessment

This assessment measures student learning after the unit is delivered. It is considered assessment OF learning and is graded. Summative assessment yields data that describes students' achievement in relation to a clearly defined set of standards. These assessments are given after students have time to practice and learn from their mistakes through formative assessment. Examples of summative assessments for a lesson are quizzes and mini-projects

STUDENT REFLECTION AND GOAL SETTING

Throughout the unit, students reflect on their work and progress then set goals for further learning. Students could keep a learning journal or file papers in their binders with reflections/goals attached to the work.

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PHASES OF THE INSTRUCTIONAL UNIT

A unit of instruction can take anywhere from a few days to a several weeks to complete. The number of a particular lesson type delivered is at the discretion of the instructor. The stages of instruction represent a complete instructional cycle.

STAGE 1: SCAFFOLDING ACCESS LESSON

Purpose

Engage students; spark curiosity; present personal connections to concepts and skills. Access refers to the minimal amount of knowledge needed by students to approach the content in the unit.

Outcome

Students connect concepts and skills to life outside the classroom and are motivated to learn. Students can answer the question, "Why am I learning this?".

Key Considerations

- Pre-Assessment of pre-requisite skills
- Broad overview of the knowledge and skills developed in the unit
- Access prior knowledge, connecting new concepts to existing schema and make interdisciplinary connections
- Varied and rigorous instructional strategies to teach content
- Content and contextual vocabulary is introduced

STAGE 2: KNOWLEDGE BUILDING LESSON

Purpose

Students explore key concepts and ways of thinking about/working with specific content skills and knowledge

Outcome

Fragile understanding of concepts and skills developed via a variety of tasks.

Key Considerations

- Introduce skills and procedures through direct instruction/guided practice and gradual release of responsibility
- Encourage students to interact with concepts in different ways
- Develop multiple representation of solutions
- Leverage collaborative learning
- Content and contextual vocabulary permeates oral and written communication

STAGE 3: PRACTICE LESSON

Purpose

Develop students' understanding of fundamental concepts through activities that engage them in classifying and defining, representing concepts in multiple ways, testing and challenging common misconceptions and exploring structure.

Outcome

Students apply strategies without prompting to problem-solving contexts and use content/contextual vocabulary accurately.

Key Considerations

- Less direct instruction continue to scaffold and/or differentiate
- Build depth and flexibility in authentic situations; use concepts across contexts
- Leverage peer collaboration
- Encourage students to try the same thing in different ways
- Use concepts across contexts

STAGE 4: FORMATIVE ASSESSMENT

Purpose
Assess and develop students' capacity to apply their mathematics flexibly to non-routine/unstructured problems, both from the real world and within pure mathematics.

Outcome
Surface students' misconceptions

Key Considerations
- Rich, non-routine tasks
- Focus on applying skills, concepts, and understandings to solve multi-step problems requiring abstract reasoning, precision, perseverance, and strategic use of tools
- Reveal a learner's understanding of a problem/task and her/his mathematical approach to it
- Could be an individual, group, or class-wide exercise

STAGE 5: RE-ENGAGEMENT LESSON

Purpose
Before the lesson, students worked individually or in groups on an assessment task designed to reveal their current understanding and difficulties. In this lesson, you

Outcome
Students move from fragile to robust understanding of concepts and skills via a variety of tasks.

Key Considerations
- Student work is presented to the class, problem by problem
- In a whole-class discussion, students compare and evaluate the validity and efficacy of strategies presented by their peers
- Supports SMP1: Construct viable arguments and critique the reasoning of others.

STAGE 6: SUMMATIVE ASSESSMENT

Purpose
Gauge student learning relative to content standards and acquisition of fluency targets.

Outcome
Use results to inform revisions to unit and lesson plans prior to the next instructional cycle. Results used to report student achievement level (grade).

Key Considerations
- Through authentic performance tasks or a project, students demonstrate what they have learned (concepts, skills, understandings, Big Idea) and can do as a result of this unit
- Individual and/or group assessment with individual accountability components

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